Features of the Acoustic Mechanism of Core-Collapse Supernova Explosions : Revisited using the Kinetic Approach

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Abstract

The discrete kinetic model is used to study the propagation of sound waves in system of hard-disk-like rotating stars (or vortex gases). The anomalous (negative) attenuation or amplification which is possibly due to the binary collision of a dilute-enough rotating disk (or vortex-gas) system (each with opposite-sign rotating direction or angular momenta but the total (net) angular momenta or vorticity is zero) or microreversibility might arise from the implicit balance of the angular momentum during encounter and give clues to the understanding of possible acceleration of cosmic rays passing through this kind of channel and direct or inverse vortex-gas cascades in two-dimensional turbulence of astrophysical problems.

Keywords : physics of the early universe, classical tests of cosmology, neutrino detectors, ultra high energy cosmic rays, core-collapse supernovas

1 Introduction

There are many proposed acoustic probes in astrophysical applications, say, studying baryon acoustic oscillations or microwave background fluctuations (due to couplings between different cosmic background particles) [1]. Recently Burrows *et al.* [2] proposed a new mechanism for core-collapse supernova explosions : the acoustic power generation in the thin core as the driver and the propagation into the mantle of strong sound waves. Acoustic power is, potentially, an efficient means to transport energy and momentum into the outer mantle to drive the supernova explosion. Unlike neutrinos, sound is almost 100% absorbed in the matter, though some of the acoustic energy is reradiated by neutrinos. As sound pulses propagate outward down the density gradient they steepen into multiple shock waves that catch up to one another and merge; a shock wave is almost a perfect black-body absorber of sound. If sufficient sound is generated in the core, it would be a natural vehicle for the gravitational energy of infall to be transferred to the outer mantle and could be the key missing ingredient in the core-collapse explosion mechanism. Using the 2D radiation/hydrodynamic code VULCAN/2D Borrows *et al.* [2] meanwhile have discovered that turbulence and anisotropic accretion in the inner 40-100 kilometers can excite and maintain vigorous core g-mode oscillations which decay by the radiation of sound. The inner core acts as a transducer for the conversion of accretion gravitational energy into acoustic power. As they used two-dimensional treatment, thus they are adopting gases of disks instead of gases of spheres [2]. In fact, there are other approaches using the kinetic or Boltzmann equations

to study neutrino physics [3-5].

Quite recently the search for ultra high energy (UHE, 10^{18-20} eV) neutrinos is strongly motivated by the observation of cosmic rays. Since the first comprehensive observations by Auger in 1938, it has been observed that by extending the coverage of detectors, increasingly higher energy events have been detected. Presently, the spectrum of these outer-space particles is known to extend up to 10^{20} eV, and maybe even higher. In particular, there is an on-going discussion on the existence, or not, of the GZK [6] cut-off on the cosmic rays spectrum. Because neutrinos interact so weakly, their observation at UHE would bring insight on the origin and nature of these UHE cosmic rays. If the GZK cut-off is relevant, it could be studied from the observation of neutrinos which will be produced as secondaries from pion disintegrations. This would signal the existence of extremely high energies cosmic particles (EHE, 10^{20+} eV). Furthermore, neutrinos are the only known candidates that would allow astrophysical observation at these extreme energies, above the GZK cut-off [8]. There already are many intensive researches reported till now [9]. Meanwhile, as reported in [10], there is a significant transformation of kinematic energy into compressible sound energy when the (superfluid Helium) vortex tangle evolves. This might imply that the production of sound is the fundamental process of dissipation of kinetic energy at absolute zero (T=0) [10]. Thus, second sound is an important tool in the research related to quantum turbulence. They can also be applied to the relevant problems in astrophysics or cosmology [11] becaus all these vortex-like studies are closely linked to system of rotating hard-disk stars. Thus, better understanding of the formation and/or decay of vorticity in the BEC and the interaction of multiple vortices in dilute Bose gases (or stirred BEC) subjected to thermal noises currently raise many challenges [12]. Either collision or shearing of vortices during encounters will generate thermodynamic noises or sound pulses. Eventually the unstable or turbulent state in BEC occurs.

In fact, a vortex could be treated as a combination of concentrated vorticity (core) with its surrounding irrotational fluid (flow) [13]. Thus, they could be, in certain sense, similar to the hard-sphere particles when the elastic scattering or collision for a system of them being in consideration [14]. Some researchers may argue that, the basic physics of vortices is, according to a brilliant formulation of Onsager [15], that configuration space and phase space coincide. So, in 2D, the x-coordinate and the y-coordinate of a vortex are conjugate variables! This has important consequences for identifying the momentum of a vortex, i.e. a vortex never behaves kinematically like a Newtonian point particle (hard sphere in 3D or hard disc in 2D). System of vortices [16-17], dilute enough and during binary encounters, however, as the chaotic behavior [18-19] resembling that of Newtonian particles upon collisions [13-14], could be treated dynamically by hard-sphere or hard-disk collisions with careful selection of the impact parameter.

First sound is well known as an ordinary sound wave and is related to a wave of pressure difference (or a kind of fluctuation in the total density) in helium II. Second sound is unique to helium II and represents a kind of fluctuation in temperature. Both share similarities to the

propagation of sound mode (first mode) and diffusion mode (second mode; entropy wave) in dilute monatomic gases or fluids which were already well captured by continuous and/or discrete kinetic models [20-22] using hard-sphere- or hard-disk-gas assumptions.

Recent Jackiw gave hydrodynamic profiles and constants of motion from d-branes and discussed supersymmetric fluid mechanics [23]. It will be interesting to further investigate other hydrodynamic properties, like sound propagation in dilute gases with similar hard-disk like collisions for nonrelativistic cases. As a preliminary attempt, considering the presumed analogy between the scattering of dilute rotating (vortex) gases [24-27] (e.g., the duality or equivalence of Bose-gas particles [24-25] and (rotating) vortices [26-27]) and dilute hard-disk gases, in this paper, we plan to investigate the dispersion relations of sound propagation in hard-disk-like rotating stars by the orientation-free discrete kinetic model which has been verified in Refs. [21-22,28]. This presentation will give some clues to the understanding of the dissipation mechanism of interactions of multiple rotating hard-disk-like stars (or vortices) and other similar problems in astrophysics or cosmology [1-9,11] (acoustic probes for two-dimensional turbulence included). We note that complicated boundary conditions are avoided by most of previous workers who did initial value problems in an unbounded domain rather than the semi-infinite problem to which measurements refer because of the essential difficulties.

In the discrete kinetic approach, the main idea is to consider that the particle velocities belong to a given finite set of velocity vectors. Only the velocity space is discretized, the space and time variables are continuous. The discrete kinetic models [21-22,28] thus come (please see the detailed references therein). For a lattice gas, the space and time variables are also discretized. By using the discrete kinetic approaches, the velocity of propagation of sound wave can be classically determined by looking for the properties of the solution of the conservation equation referred to the Maxwellian state. Theoretical attempts link quantum-mechanic to Boltzmann approach have been well developed (see, e.g., [22]) which provide us more applications for the present approach.

In this presentation, we shall introduce our theoretical approach in the Section 2. The numerical results and discussions will be put in the final Section. We shall firstly present those fixed-orientation results and then other remaining free-orientation results which might be useful to interpret those claims in [2].

2   Theoretical Formulations

We make the following assumptions before we investigate the general equations of our model :

(1) Consider a gas of identical particles of unit mass and a shape of a disk of diameter d, then each particle i, $i = 1, \cdots, N$, is characterized by the position of its center q_i and its velocity v_i. We also have the geometric limitations : $|q_i - q_j| \geq d$, $i \neq j$.

(2) Each particle moves in the plane with velocity belonging to a discrete set $\mathcal{V}$ of 4 velocities

with only one speed in the plane. The velocity modulus c is a reference speed depending on the reference frame and specific distribution of particles. c is normally linked to the internal energy of the molecules in thermodynamic equilibrium (please see Fig. 2).

(3) The collisional mechanism is that of rigid spheres, that is, the particles scatter elastically and they change their phase states instantaneously, preserving momentum. Only binary collisions are considered, since a multiple collision here is a negligible events.

The collisions between two particles (say i and j) take place when they are located at q_i and $q_j = q_i - d\mathbf{n}$, where $\mathbf{n}$ is the unit vector joining their centers. After collisions the particles scatter, preserving momentum, in the directions allowed by the discrete set $\mathcal{V}$. In other words, particles change according to

$$(q_i, v_i) \to (q_i, v_i^*), \qquad (q_j, v_j) \to (q_j, v_j^*).$$

The collision is uniquely determined if the incoming velocity and the impact angle ψ, $\psi \in [-\pi/2, \pi/2]$, are known, which is defined as the angle between v_i and $\mathbf{n}$ or $\mathbf{n}(\psi) = (\cos[\psi + (k - 1)\pi/2], \sin[\psi + (k - 1)\pi/2])$, $k = 1, \cdots, 4$.

From the selected velocities we have two classes of encounters, i.e. $\langle v_i, v_j \rangle = 0$ and $\langle v_i, v_j \rangle = -c^2$, respectively.

(a). In the first class momentum conservation implies only : encounters at $\pi/2$ with exchange of velocities

$$v_i = v^k \to v_i^* = v^{k+1}, \qquad v_j = v^{k+1} \to v_j^* = v^k, \quad k = 1, \cdots, 4,$$

in the case $\psi \in [-\pi/2, 0]$, and

$$v_i = v^k \to v_i^* = v^{k+3}, \qquad v_j = v^{k+3} \to v_j^* = v^k,$$

in the case $\psi \in [0, \pi/2]$.

(b) Similarly, $\langle v_i, v_j \rangle = -c^2$;

(i) Head-on encounters with impact angle $\psi = 0$ such that

$$v_i = v^k \to v_i^* = v^{k+2}, \qquad v_j = v^{k+2} \to v_j^* = v^k, \quad k = 1, \cdots, 4,$$

(ii) Head-on encounters with impact angle $\psi \neq 0$ such that

$$v_i = v^k \to v_i^* = v^{k+1}, \qquad v_j = v^{k+2} \to v_j^* = v^{k+3}, \qquad \text{if } \psi \in [-\pi/2, 0],$$

$$v_i = v^k \to v_i^* = v^{k+3}, \qquad v_j = v^{k+2} \to v_j^* = v^{k+1}, \qquad \text{if } \psi \in [0, \pi/2].$$

For grazing collisions, that is $\langle \mathbf{n}, v_i \rangle = \langle \mathbf{n}, v_j \rangle = 0$, we put $v_i^* = v_i$, $v_j^* = v_j$. Schematic presentation is illustrated in Figs. 1 and 2.
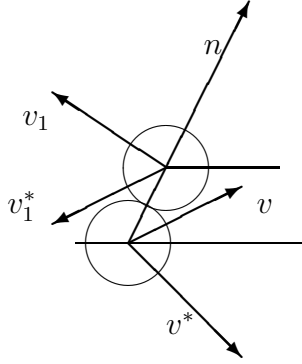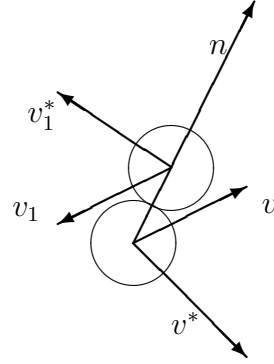
Fig. 1   Schematic (plot) of a binary collision.   Fig. 2   A *head-on* collision.

The verification of our approach with the previous available approaches (propagation of forced sound-mode) has been done in Refs. [21-22]. Here, we only consider the one-dimensional propagation of plane wave by neglecting the complicated real boundary conditions.

We assume that the system of hard-disk gas (star) is composed of identical particles (stars) of the same mass. The velocities of these particles are restricted to, e.g., : $\mathbf{v}_1, \mathbf{v}_2, \cdots, \mathbf{v}_p$, p is a finite positive integer. The discrete number density of particles are denoted by $N_i(\mathbf{x}, t)$ associated with the velocity $\mathbf{v}_i$ at point $\mathbf{x}$ and time t.

This simplified model, i.e., the $2 \times n$-velocity model, is to consider a one-component discrete velocity (rotating) disk- (or vortex-)gas such that the particles can attain $2n$ velocities in the 2D-plane. In particular, the velocity discretization is characterized by

(i) $|\mathbf{v}_i| = c$,

(ii) $\mathbf{v}_i + \mathbf{v}_{i+n} = 0$,

(iii) $\mathbf{v}_i \cdot \mathbf{v}_{i+1} = c^2 \cos(\pi/n)$, $i = 1, \cdots, 2n$;

where the index is to be intended modulo $2n$, i.e. $i \equiv i + 2n$. Such a model is called the planar $2n$-velocity model. If only elastic collisions are taken into account, then the non-trivial admissible ones (where this term is used to denote those collisions which produce non-vanishing terms in the collision operator) are [21-22]

$$(\mathbf{v}_i, \mathbf{v}_{i+n}) \longleftrightarrow (\mathbf{v}_j, \mathbf{v}_{j+n}) \qquad \forall j \neq i, i = 1, \cdots, 2n.$$

Besides, the momentum and energy are presumably preserved

$$\mathbf{v}_i + \mathbf{v}_{i+n} = \mathbf{v}_j + \mathbf{v}_{j+n},$$

$$|\mathbf{v}_i|^2 + |\mathbf{v}_{i+n}|^2 = |\mathbf{v}_j|^2 + |\mathbf{v}_{j+n}|^2.$$

Moreover, all the velocity directions after collisions are assumed to be equally probable. We have no intentions to consider those unstable encounter/departure for two-(rotating) vortex (or

disk) collisions [17-18,29] (please see also the references therein).

Considering binary (two-disk encounter each time) collision only, the equation of discrete kinetic models proposed in Refs. [21-22,28] is a system of $2n(=p)$ semilinear partial differential equations of the hyperbolic type :

$$\frac{\partial}{\partial t}N_i + \mathbf{v}_i \cdot \frac{\partial}{\partial \mathbf{x}}N_i = \frac{2cS}{n}\sum_{j=1}^{n}N_jN_{j+n} - N_iN_{i+n}, \qquad i = 1,\cdots,2n, \tag{1}$$

where $N_i = N_{i+2n}$ are unknown functions, and $\mathbf{v}_i = c(\cos[\theta + (i-1)\pi/n], \sin[\theta + (i-1)\pi/n])$; c is a reference velocity modulus, S is an effective collision cross-section for the 2-(rotating)disk system [21-22,28], θ is the free orientation parameter (the orientation starting from the positive x-axis to the v_1 direction and is relevant to the (net) induced scattering measured relative to the sound-propagating direction) which might be linked to the external field or the angular momentum or the rotation effects.

Since passage of the sound wave causes a small departure from equilibrium (Maxwellian type) resulting in energy loss owing to internal friction and heat conduction, we linearize above equations around a uniform Maxwellian state (N_0) by setting $N_i(t,x) = N_0(1 + P_i(t,x))$, where P_i is a small perturbation. The Maxwellian here is presumed to be the same as in Refs. [20-21]. After some manipulations we then have

$$[\frac{\partial^2}{\partial t^2} + c^2\cos^2[\theta + \frac{(m-1)\pi}{n}]\frac{\partial^2}{\partial x^2} + 4cSN_0\frac{\partial}{\partial t}]D_m = \frac{4cSN_0}{n}\sum_{k=1}^{n}\frac{\partial}{\partial t}D_k, \tag{2}$$

where $D_m = (P_m + P_{m+n})/2$, $m = 1,\cdots,n$, since $D_1 = D_m$ for $1 = m$ (mod $2n$). We are ready to look for the solutions in the form of plane wave $D_m = a_m \exp i(kx - \omega t)$, $(m = 1,\cdots,n)$, with $\omega = \omega(k)$. This is related to the dispersion relations of 1D forced ultrasound propagation of dilute monatomic hard-sphere gases problem. So we have

$$\{1 + ih - 2\lambda^2\cos^2[\theta + \frac{(m-1)\pi}{n}]\}a_m - \frac{ih}{n}\sum_{k=1}^{n}a_k = 0, \qquad m = 1,\cdots,n, \tag{3}$$

where $\lambda = kc/(\sqrt{2}\omega)$, $h = 4cSN_0/\omega \propto 1/\text{K}_n$ is the rarefaction parameter of the gas; K_n is the Knudsen number which is defined as the ratio of the mean free path of hard-disk or vortex gases to the wave length of the plane sound wave .

Let $a_m = \mathcal{C}/(1 + ih - 2\lambda^2\cos^2[\theta + (m-1)\pi/n])$, where $\mathcal{C}$ is an arbitrary, unknown constant, since we here only have interest in the eigenvalues of above relation. The eigenvalue problems for different $2 \times n$-velocity model reduces to $F_n(\lambda) = 0$, or

$$1 - \frac{ih}{n}\sum_{m=1}^{n}\frac{1}{1 + ih - 2\lambda^2\cos^2[\theta + \frac{(m-1)\pi}{n}]} = 0. \tag{4}$$

We solve $n = 2$ case, i.e., 4-velocity case. The admissible collision : $(1,3) \longleftrightarrow (2,4)$ for system of rotating disks or vortex gases during binary encounter is shown schematically in Fig. 3. The corresponding eigenvalue equations become algebraic polynomial-form with the complex roots being the results of λs.

For 2×2-velocity model, we obtain $\quad 1 - (ih/2) \sum_{m=1}^{2} \{1/(1 + ih - 2\lambda^2 \cos^2[\theta + (m-1)\pi/2])\} = 0$.

3   Numerical Results and Discussions

By using the standard mathematical or numerical software, e.g. Mathematica or Matlab, we can obtain the complex roots ($\lambda = \lambda_r + i\,\lambda_i$) for the polynomial equations above [21-22]. The roots are the values for the nondimensionalized dispersion (positive real part) and the attenuation or absorption (positive imaginary part), respectively. We plot those of $\theta = 0$ into Fig. 4. Curves of branch I follows the conventional dispersion relation of ultrasound propagation in dilute monatomic hard-sphere gases [21-22]. Our results could also be applied to the case : as one rotating disk (star or vortex) colliding with the plane boundary (the image disk (star or vortex) moving in the opposite direction within the boundary domain and approaching to the boundary too [17-18,29]) after then, i.e. the at-a-distance encounter with the impact parameter being small but finite, they depart (a grazing collision [30]) following the routes perpendicular to their original direction.

From a modern point of view, dissipations of the sound wave arise fundamentally because of a necessary coupling between density and energy fluctuations induced by disturbances. Within one mean free path or so of an oscillating boundary, a free molecular flow solution can probably be computed. The damping will quite likely turn out to be linear because the damping mechanism is the shift in phase of particles which hit the wall at different times. It seems there is a classical (dynamical) wave-localization around $h \sim 1$ (cf. (Chu, 2001) in [21]). To conclude for the results of sound mode, it was observed that, whereas the continuum-mechanic approach provides a good modeling at low frequencies, it is definitely not adequate at high frequencies $h \leq 2$. Especially the zero dispersion (phase speed) as h approaches zero [21-22]. As the wavelength of sound is made significantly shorter, so that the effects of viscosity and the heat conduction are no longer small, the validity of continuum-mechanic approach itself becomes questionable. If there is no rarefaction effect ($h = 0$), we have only real roots for all the models [21-22]. Once $h \neq 0$, the imaginary part appears and the spectra diagram for each model looks entirely different. In short, the dispersion ($k_r c/(\sqrt{2}\omega)$) reaches a continuum-value of 1 for the 4-velocity models once h increases to infinity. The attenuation ($k_i c/(\sqrt{2}\omega)$) for the same models, instead, firstly increases up to $h \sim 1.7$, then starts to decrease as h increases furthermore.

Curves of branch II, however, show a entirely different trend. The dispersion part seems to follow the diffusion mode reported in Refs. [21,31]. It increases but never reaches to a limit. The anomalous attenuation might be due to, if any, the intrinsic resonance (an *eigen-oscillation*) [32] or the implicit behavior of angular momentum relation during 2-(rotating)disk (or vortex) encounter (each with opposite-sign rotating direction or vorticity or angular momenta so that the total (net) vorticity or angular momenta for this two-body encounter is zero) since the latter

is absent or of no need in the formulation of 2-body collisions. We note that the vorticity or direction of the rotational axis of each disk or vortex (ready to encounter) for the collision of 2-(rotating)disk (or vortex) system might be in opposite sign instead of the same sign. We don't know yet at present whether the former or the latter can favor the anomalous attenuation ? However, the cosmic ray passing through this acoustic-amplified channel might be accelerated (neutrinos included)!

Some researchers argued that this anomalous phenomenon (negative attenuation or amplification) might be linked to the direct or inverse vortex cascades (in two-dimensional turbulence). Note that, another possible explanation is due to the presumption of the *microreversibility* in the formulation of equation (1) [21-22,28]. This assumption is valid for the reverse collision in the phase space and is common in the formulation of the Boltzmann approach [21-22,28,30]. There is a newest (numerical) report [33] about the physical reasons for amplification of sound waves which are discussed in terms of redistribution of acoustic energy and its potential and kinetic components. The role of acoustic energy exchange with the mean flow was investigated therein. Compressibility of the background mean flow is taken into account and its effect on amplification of acoustic pressure was discussed. All these just mentioned above could also be valid to the system of rotating (hard-disk-like) stars!

The results presented here also show the intrinsic thermodynamic properties of the presumed equilibrium states corresponding to the collision of rotating disks or vortex gases during binary encounter [34-35]. The rigorous proof of their existence, however, will be our future work.

To check the free-orientation effects, we plot those results in Fig. 5. We can observe, the smaller (absolute values of λ) branch (propagation of sound mode) or lower values of λ_i in this figure which shows a continuous trend as θ increases toward $\pi/4$. The acoustical attenuation or absorption keeps decreasing as θ increases from 0. At $\theta = \pi/4$, there is no attenuation, i.e. $\lambda_i = 0$. Based on this final orientation-free results, we could make remarks about those presented in [2] : the angular distribution of the emitted sound is fundamentally aspherical once there are rotation, accretion and/or magnetic fields existing during their propagation (of plane sound waves). The diffusion modes observed in Fig. 4 or Refs. [21,31] might also be crucial to the sound pulses radiated from the core which steepen into shock waves that merge as they propagate into the outer mantle and deposit their energy and momentum with high efficiency [2]. We shall investigate other relevant problems [36] and role of disk-searching (linked to massive star formation) [37] in the future.

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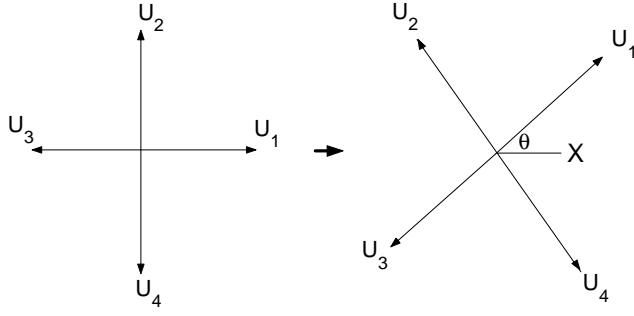C.F. McKee and J.C. Tan, Nature **416**, 59 (2002).

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Fig. 3   Schematic plot for the regular scattering and the orientational scattering.
Plane waves propagate along the X-direction. Binary encounters of U_1 and U_3 and their
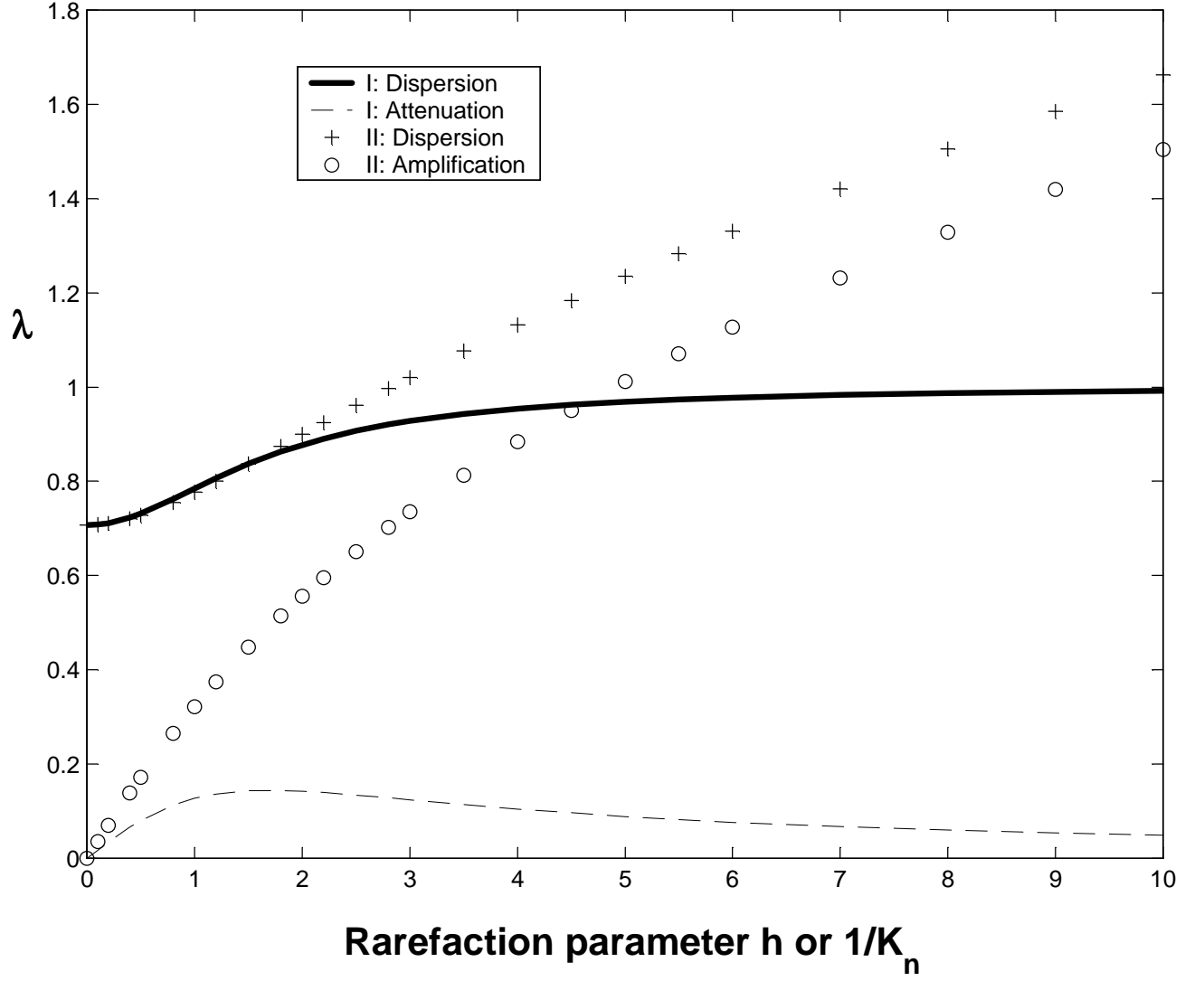departures after head-on collisions (U_2 and U_4). Number densities N_i are associated to U_i.

Fig. 4   Dispersion relations of branches I & II over a range of h (rarefaction measure). λ_r : phase speed dispersion, λ_i : attenuation or amplification. $\lambda = \lambda_r + \mathrm{i}\,\lambda_i$. $\theta = 0$ here.
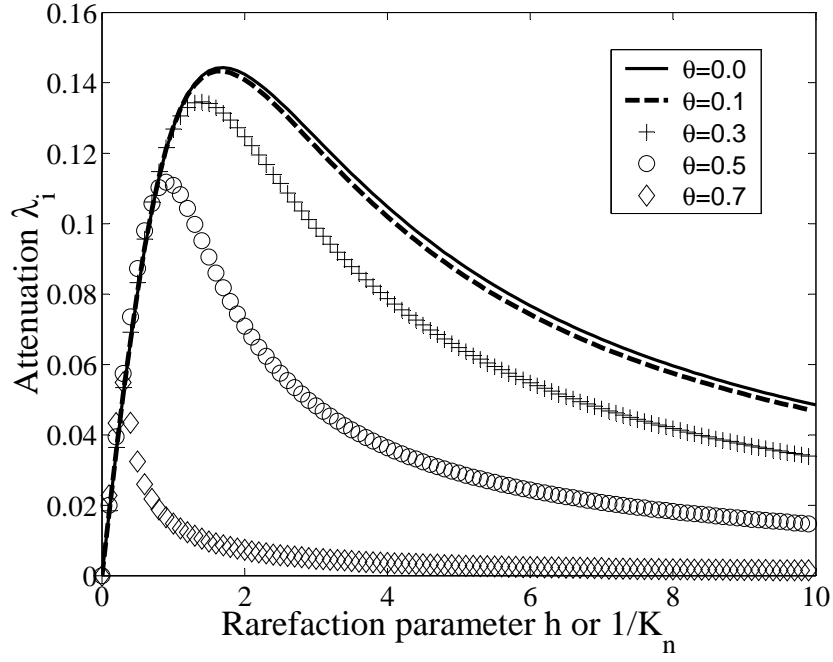
Fig. 5   Free-Orientation effects on the attenuation or absorption of acoustic waves (λ_i). This free orientation might be due to the external field or rotational effect (relevant to the (net) angular momenta existing during the 2-(rotating)disk encounter).